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This is a summary of research completed during the period of this award, 1 October 1985-30 September 1988 by Fleming, Kushner together with associated postdoctoral and graduate student personnel. The research covers a number of problems in many areas of stochastic control, recursive stochastic algorithms, and related areas of analysis. It is part of a continuing research program pursued successfully for a number of years. The program has been motivated both by traditional applications in control and filtering and by newer areas of application arising in queueing/communication networks and production systems. Other research issues addressed include numerical methods for stochastic control and recursive algorithms for distributed and paallel processing and/or control.
The work of Fleming and Kushner will be summarized in turn, with references to research publications supported under this award.

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"NUMERICAL METHODS AND APPROXIMATION AND MODELLING PROBLEMS IN STOCHASTIC CONTROL THEORY."

Principal Investigators:

Wendell H. Fleming
Harold J. Kushner

This is a summary of research completed during the period of this award, 1 October 1985 - 30 September 1988 by Fleming, Kushner together with associated postdoctoral and graduate student personnel. The research covers a number of problems in many areas of stochastic control, recursive stochastic algorithms, and related areas of analysis. It is part of a continuing research program pursued successfully for a number of years. The program has been motivated both by traditional applications in control and filtering and by newer areas of application arising in queueing/communication networks and production systems. Other research issues addressed include numerical methods for stochastic control and recursive algorithms for distributed and parallel processing and/or control.

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W. H. Fleming

Fleming's research was concerned with optimal stochastic control theory, nonlinear filtering, large deviations for Markov diffusions, and viscosity solutions of nonlinear partial differential equations.

Fleming's work in optimal stochastic control focused on control of Markov diffusion processes, with complete or partial observations. This work is reported in references [F3] [F4] [F5] [F11 - F12]. [F3] [12] provide a solution to a stochastic production planning problem, making use of viscosity solution methods for a system of first-order nonlinear partial differential equations. [F4] [F11] provide an approximate solution, in the form of an asymptotic series in powers of a small parameter measuring the intensity of noise entering the control dynamics. References [F5] [F13] provide an alternative to the usual dynamic programming, seeking maximal smooth subsolutions rather than solutions to the dynamic programming equations and using an abstract duality theorem from convex analysis.

The theory of large deviations is concerned with asymptotic estimates for exponentially small probabilities of rare events associated with stochastic processes. In 1977, Fleming introduced a stochastic control approach to large deviations of nearly deterministic Markov diffusions. This technique was based on a certain logarithmic transformation. Subsequently, simpler analytical methods based on logarithmic transformations and viscosity solution techniques were developed [F1]. These methods also give more accurate

approximations in the form of an asymptotic series [F4][F11]. These analytical methods were adapted to other classes of Markov processes in [F8].

Fleming and Souganidis [F7][F14] developed a theory of value for two-player, zero sum stochastic differential games. This provides an optimization formula for viscosity solutions to a large class of nonlinear second partial differential equations, of degenerate parabolic type.

References [F9] [F10] [F15] are concerned with nonlinear filtering, in cases when a namy-to-one function function of the system state plue low intensity observation noise is observed. The goal is to obtain finite-dimensional, approximately optimal filters.

Research Publications W.H. Fleming

[F1] - [F15]

F 1. A PDE Approach to Asymptotic Estimates for Optimal Exit Probabilities (with P.E. Souganidis), Annali della Scuola Normale Superiore Pisa, Ser. IV 23 (1986) 171-192.

F 2. Stochastic Variational Formula for Fundamental Solutions of Parabolic PDE (with S-J Sheu), Applied Math and Optimization, 13 (1985).

F 3. An Optimal Stochastic Production Planning Problem with Randomly Fluctuating Demand, (with S.P. Sethi and H.M. Soner), SIAM J. on Control and Optimization, 25 (1987). 1494-1502.

F 4. Asymptotic Series and the Method of Vanishing Viscosity (with P.E. Souganidis), Indiana Univ. Math. J. 35 (1986) 425-447.

F 5. Convex Duality Approach to the Optimal Control of Diffusions (with D. Vermes), SIAM J on Control and Optimiz to appear in 1989.

F 6. On the Existence of the Dominant Eigenvalue and its Application to the Large Deviation Properties of an Ergodic Markov Process (with S. J. Sheu and H.M. Soner), 22 (1987) 187-199.

F 7. Value Functions for Two-Player, Zero-Sum Stochastic Differential Games (with P.E. Souganidis), Indiana Univ. Math J, to appear in 1989.

F 8. Asymptotic Expansions for Markov Processes with Levy Generators (with H. M. Soner), submitted to Applied Math. and Optimiz, 19 (1989) 203-223.

F 9. Piecewise monotone filtering with small observation noise (with E. Pardoux), SIAM J. on Control and Optimiz (1989).

F 10. Discrete time piecewise linear filtering with small observation noise (with D. Ji, P. Salame, Q. Zhang) LCDS/CCS Report No. 88-27.

F 11. Asymptotic Series for Solutions to the Dynamic Programming Equation for Diffusions with Small Noise, (with P.E. Souganidis) Proc. 24th IEEE Conf. on Decision and Control, Ft. Lauderdale, Florida, December 11-13, Vol. 1, 1985.

F 12. A Stochastic Production Planning Problem with Random Demand, (with H.M. Soner) Proc. 24th IEEE Conf. on Decision and Control, Vol. 1, Ft. Lauderdale, Florida, December 11-13, 1985.

F 13. Generalized Solutions in the Optimal Control of Diffusions (with D. Vermes) Proc. IMA Workshop, June 1986, IMA Vols. in Math. and Applic. No. 10, Springer-Verlag, 1987, 119-127.

F 14. Two-Player, Zero-Sum Stochastic Differential Games (with P.E. Souganidis) Proc. of Conf. in honor of J.L. Lions, June 1988.

F 15. Piecewise-linear filtering with small observation noise (with D. Ji and E. Pardoux) Proc. 8th INRIA Conf. on Analysis and Optimiz. of Systems, Springer Lect. Notes in Control and Info. Sci: No. 111, 1988.

H.J. Kushner

Kushner's research covered a wide range of topics in stochastic systems theory and applied probability. These including: large deviations with communications applications, stochastic approximations (convergence theorems, large deviations estimates), adaptive filters, distributed parameter stochastic systems, wide band noise approximations, Monte Carlo methods, distributed and communicating stochastic approximation algorithms, singular stochastic control and computational methods in optimal stochastic control.

The work on large deviations and applications is reported in [K4] [K8] [K16] [K17]. Typical communications applications arise in models with rapidly varying noise inputs, for slowly adapting digital systems, and for tracking systems with small noise effects. Among the accomplishments is a "quick simulation"technique, based on a change of probability measure technique. This method relies on the numerical solution of a first-order nonlinear partial differential equation, connected with the action functional being minimized to obtain the large deviation rate.

The papers on stochastic approximation and recursive algorithms [K5] [K10] [K11] [K21] provide limit theorems and large deviations estimates under conditions on dynamics and noise which are broad enough to fit most current applications in control and communication theory. The global behavior of stochastic approximations was studied by Monte Carlo methods in [K7].

Kushner's work on distributed parameter stochastic systems, described by stochastic partial differential equations, is reported in [K2] [K3]. This work is concerned with such questions as stability and near-stationarity for systems with wide-band noise inputs, and with nonlinear filtering applications.

Further work on wide-band noise approximations in filtering and stochastic control is reported in [K6] [K9] [K14] [K15] [K24]. In the part of this work concerned with stochastic control, the problem is to find nearly-optimal control laws for a wide-band noise driven system, based on optimal control laws for an idealized white-noise driven system (for which there is an extensive theory.)

Recent work by Kushner on stochastic approximation [K12] [K13] concerns distributed and communicating systems, in which processers are located at physically distinct sites.

Routing and flow control problems for queues under heavy traffic can be analyzed using methods of singular stochastic control. Such problems, and related problems for wide-band noise driven singular stochastic control systems were analyzed in [K18] [K19][20]

Ref. [22] is an invited survey of recent developments on numerical methods for stochastic control problems. It complements Kushner's earlier book which dealt with this topic "Probability Methods for Approximation in Stochastic Control and for Elliptic Equations ", 1977.

Research Publications - H.J. Kushner

K 1. Approximating multiple Ito integrals with "band-limited" processes, Stochastics, 14, 1985, pp 85-114.

K 2. Weak convergence approximations for partial differential equations with stochastic coefficients, Stochastics, 14, 1985, pp 115-148.

K 3. Asymptotic properties, stability and "near" stationanity of parabolic partial differential equations with wide bandwidth inputs, Stochastics, 16, 1986, pp 111-136.

K 4. Large deviations estimates for systems with small noise effects, and applications to stochastic systems theory, (with P. DuPuis) SIAM J. on Control and Optimization, 24, 1986, pp 979-1008.

K 5. The theory of large deviations and asymptotic analysis of recursive algorithms and stochastic approximation (with P. DuPuis), in Advances in Statistical Signal Processing, ed. by H.V. Poor, JAI Press.

K 6. Approximate and limit results for nonlinear filters with wide bandwidth observation noise, Stochastics, 16, 1986, pp 65-96.

K 7. Asymptotic global behavior for stochastic approximations with slowly decreasing gain: Global optimization via Monte-Carlo, in SIAM J. on Appl. Math., 47, 1987, 169-185.

K 8. Stochastic systems with small noise, analysis and simulation, a phase locked loop example, SIAM J. Appl. Math., 47, 1987, 643-661.

K 9. Nearly optimal state feedback controls for stochastic systems with wide-band noise disturbances, (with W. Runggaldier) SIAM J. Control and Optimization, 25, 1987, 298-315.

K 10. Constrained stochastic approximation by the theory of large deviations, 'Robbins Symposium Volume', published by Wiley.

K 11. Asymptotic behavior of constrained stochastic approximations via the theory of large deviations. Z. Wahrscheinlichkeitsteorie, 75,1987, p 224-244.

K 12. Asymptotic properties of distributed and communicating stochastic approximation algorithms (with G. Yin), in SIAM J. Control and Optimization, 25, 1987; 1266-1290.

K 13. Stochastic approximation algorithms for parallel and distributed processing (with G. Yin), Stochastics, 22, 1987, p 219-250.

K 14. Filtering and control for wide bandwith noise driven systems IEEE Trans. on Automatic control, T-AC87, 1987, p 123-133.

K 15. Almost optimal controls for wide band noise driven systems, Institute of Math. and Appl. (Minnesota) Volume 10, pub. by Springer, 1987 (W. Fleming, P.L. Lions, ed.).

K 16. Minimizing escape probabilities: a large deviations approach (with P. DuPuis) to appear SIAM J. on Control and Optimization.

K 17. Upper bounds for large deviations for non-smooth stochastic difference eqns., LCDS Dept. 87-8 (Feb. 87) sub. to App. Prob.

K 18. Nearly optimal singular controls for wideband noise driven systems, (with K. M. Ramachandran), SIAM J. on Control and Optimization [26], p 569-591, 1988.

K 19. Optimal and approximately optimal control policies for queues in heavy traffic (with K.M. Ramachandran) to appear SIAM J. on Control and Optimization.

K 20. Diffusion approximations and nearly optimal maintenance policies for system breakdown & repair policies, to appear in Appl. Math. and Optimization.

K 21. Stochastic approximation and large deviations: General results for w. p. 1 convergences, (with P. DuPuis) to appear SIAM J. on Control and Optimization, LCDS Dept. 87-21, 1987.

K 22. Numerical methods for stochastic control problems in continuous time, invited paper SIAM J. on Control and Optimization.